Solve 3rd-order diff eq. that's not factorable into 3 factors: (#7 in MML)

Solve $y''' + 7y'' + 2y' - 10y = 0$

aux eqn: $r^3 + 7r^2 + 2r - 10 = 0$

$\cancel{r^2(r+7) + 2(r-5)} = 0$

Ah! Can't factor!

Solns will be
$y = e^{r_1 t}$
$y = e^{r_2 t}$
$y = e^{r_3 t}$

Solve:

Graph: $y = x^3 + 7x^2 + 2x - 10$

(Look for integer root.)

Looks like $x = 1$ is a root.
verified ✓

So, $x-1$ is a factor of $x^3 + 7x^2 + 2x - 10$.

divisor → $x-1$

$$
\begin{array}{r}
x^2 + 8x + 10 \\
x-1 \,\big)\, \overline{x^3 + 7x^2 + 2x - 10} \text{ dividend} \\
-(x^3 - x^2) \\
\hline
8x^2 + 2x \\
-(8x^2 - 8x) \\
\hline
10x - 10 \\
-(10x - 10) \\
\hline
0
\end{array}
$$

So,
$x^3 + 7x^2 + 2x - 10$
$= (x-1)(x^2 + 8x + 10)$

So, $r^3 + 7r^2 + 2r - 10 = 0$
$(r-1)(r^2 + 8r + 10) = 0$

$r^2 + 8r + 10 = 0$

$r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-8 \pm \sqrt{64 - 4(1)(10)}}{2}$

$r = \dfrac{-8 \pm \sqrt{24}}{2} = \dfrac{-8 \pm 2\sqrt{6}}{2} = -4 \pm \sqrt{6}$

$\boxed{\text{Roots: } r_1 = 1,\ r_2 = -4 + \sqrt{6},\ r_3 = -4 - \sqrt{6}}$

Gen soln: $y = c_1 e^{1t} + c_2 e^{(-4+\sqrt{6})t} + c_3 e^{(-4-\sqrt{6})t}$